### SELECTIVE MASS TRANSFER IN A MEMBRANE ABSORBER

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A theoretical study of selective mass transfer in a plane-frame membrane absorber (contactor) has been made. A mathematical model of the process has been developed and the process of purification of a gas mixture depending on the flow parameters, the membrane, and the feeding-mixture composition has been studied with its help.

The membrane absorber, or membrane contactor (MC), in which the process of selective absorption of gasphase components by the liquid through the membrane is realized, makes it possible to realize a high selectivity of the process characteristic of absorption in a wide range of operating parameters, which is inherent in the membrane method [1-7].

Both porous and nonporous membranes can be used in the MC. Usually, if possible, high-permeability porous membranes are used, although in so doing the differential pressure depending on the degree of wetting of the membrane with the liquid carrier is limited on the membrane [1, 2]. Nonporous membranes have considerably lower values of permeabilities, but under certain, practically interesting operating conditions of contactors the membrane capacity has a weak effect on the process [3].

It is expedient to make a choice of optimal parameters of the MC and operating conditions for each particular problem on the basis of numerical experiments with the use of mathematical models that most fully describe the processes proceeding in the MC. Usually the methods for calculating mass transfer in the membrane contactor are based on the use of empirically determined macroscopic mass-transfer characteristics [1, 2, 5, 8, 9]. However, they are applicable only in a narrow domain of variability of parameters, do not permit predetermination of constructive solutions and optimal operating conditions, and in many cases lead to incorrect results [2, 10]. A more general description of the mass-transfer process at the macroscopic level is the description based on the use of local mass, momentum, and energy balance equations. Such an approach should include a description of the transfer processes in the gas, liquid, and membrane phases [10]. The mass transfer in the gas phase of the membrane module has been studied fairly well, and all basic effects can be taken into account in the mathematical description [11–13].

In the liquid phase of the MC, simulation of the mass-transfer process has been developed fairly well only for the special case of the absence of concentration boundary layers in the liquid [14, 15]. In membrane separation of gases such a regime is called ideal displacement [16]. However, these assumptions hold only in the case of small liquid flows, which sharply narrows the field of their application.

In [10], a model of mass transfer in the liquid phase based on local balance equations of components in the liquid was considered. This model can be used in a wide domain of variability of hydrodynamic parameters of the liquid flow. However, it was formulated for particular cases of simplified conditions at the interface at constant concentrations of components in the gas phase.

Correct description of the mass-transfer process in the MC calls for a mathematical model including interrelated equations of mass transfer in the gas, liquid, and membrane phases. The present paper considers a model of mass transfer of a multicomponent mixture in the MC based on the use of the two-dimensional approximation of the mass balance equations of components in the liquid phase with account for interconversions of components and the one-dimensional approximation of the mass and momentum balance equations interrelated by the equations of transfer in the membrane with account for the conditions at the interface. The model is applicable for describing the mass-transfer

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Fig. 1. Diagram of the plane-frame membrane contactor.

process in MCs with porous and nonporous membranes and adsorbents of any type (arbitrary number of sorption channels) in a wide domain of variability of the operating parameters.

**Mathematical Model of the Process.** The plane-frame membrane contactor (Fig. 1) consists of gas and liquid channels, with counterflowing gas and liquid. Consider an isothermal laminar flow of a homogeneous liquid phase in a clearance of small thickness when the mass balance equation of components in the liquid is written in the form [17, 18]

$$V\frac{\partial c_j}{\partial x} = D_j \frac{\partial^2 c_j}{\partial y^2} + \sigma_j .$$
<sup>(1)</sup>

The flow velocity of the liquid in a flat slot can determined from the Poiseuille equation [19]:

$$V = \frac{6L}{Hb_{\text{lig}}^3} y \left( b_{\text{lig}} - y \right).$$
<sup>(2)</sup>

It should be noted that in the case of large concentrations of sorbed components in the liquid the diffusion coefficients depend on the concentrations  $D_j = D_j$  ( $c_1, c_2, ..., c_Z$ ) and the change in the volumetric rate of flow L along the channel length is determined by the dependence of the liquid carrier density on the concentrations  $\rho = \rho(c_1, c_2, ..., c_Z)$ . In the general case, the source function of the *j*th component  $\sigma_j$  takes into account not only the component's sinks and sources at the cost of chemical reactions, but also any other interconversion of components differing in the set of transfer coefficients. In other words, even in the case of physical sorption of a pure gas in the liquid a whole number of components, e.g., associates with a different number of functional groups, which will differ in the values of diffusion coefficients, can be formed. Therefore, it is necessary to solve Eq. (1) for each of them with account for their interconversions.

Let us assume that in the liquid all conversions of components  $A_i$  can be described by means of the equations of reactions expressed in terms of the first- and second-order elementary stages:

$$A_j \leftrightarrow A_m, \ j = 1, Z, \ m = 1, Z, \ m \neq j;$$
(3)

$$A_j + A_k \leftrightarrow A_m, \quad j = \overline{1, Z}, \quad k = \overline{1, Z}, \quad m = \overline{1, Z}, \quad m \neq j, \quad m \neq k.$$

$$\tag{4}$$

Then the source function takes on the form

$$\sigma_{j} = \sum_{k=1}^{Z} \kappa_{1kj}^{+} a_{k} - \sum_{k=1}^{Z} \bar{\kappa_{1kj}} a_{j} + \sum_{p=1, p \neq j}^{Z} \sum_{k=1, k \neq j}^{Z} \kappa_{2pkj}^{+} a_{p} a_{k} - \sum_{p=1, p \neq j}^{Z} \sum_{k=1, k \neq j}^{Z} \bar{\kappa_{2pkj}} a_{j} + \sum_{k=1, k \neq j}^{Z} \sum_{m=1, m \neq j}^{Z} \bar{\kappa_{2jkm}} a_{m} - \sum_{p=1, p \neq j}^{Z} \bar{\kappa_{2pkj}} a_{p} a_{k} - \sum_{p=1, p \neq j}^{Z} \bar{\kappa_{2pkj}} a_{j} a_{j} + \sum_{k=1, k \neq j}^{Z} \sum_{m=1, m \neq j}^{Z} \bar{\kappa_{2jkm}} a_{m} - \sum_{p=1, p \neq j}^{Z} \bar{\kappa_{2pkj}} a_{j} a_{k} - \sum_{p=1, p \neq j}^{Z} \bar{\kappa_{2pkj}} a_{j} a_{j}$$

$$-\sum_{k=1,k\neq j}^{Z}\sum_{m=1,m\neq j}^{Z}\kappa_{2jkm}^{+}a_{j}a_{k} + \sum_{p=1,p\neq j}^{Z}\sum_{m=1,m\neq j}^{Z}\kappa_{2pjm}^{-}a_{m} - \sum_{p=1,p\neq j}^{Z}\sum_{m=1,m\neq j}^{Z}\kappa_{2pjm}^{+}a_{p}a_{j}$$

where  $\kappa_{1kj}^+$  and  $\bar{\kappa}_{1kj}$  are constants of the direct and inverse reactions (3) of the *j*th-component formation from the *k*th component;  $\kappa_{2jkm}^+$  and  $\bar{\kappa}_{2jkm}^-$  are constants of the direct and inverse reaction (4) of *m*th-component formations in the reaction of the *j*th and *k*th components.

If reaction (3) or (4) does not take place at some combination of coefficients j, k, and m, we will consider the corresponding reaction rates to be equal to zero. From the problem symmetry one of the boundary conditions for (1) is written as follows:

$$\left. \frac{\partial c_j}{\partial y} \right|_{y=b_{yy}/2} = 0.$$
<sup>(5)</sup>

Another condition can be obtained by considering the mass balance at the membrane-liquid interface:

$$-D_j \frac{\partial c_j}{\partial y} = j_{j\text{m-liq}} \,, \tag{6}$$

where  $j_{jm-liq}$  is the component flow from the interface into the liquid. The conditions at the inlet into the MC of the liquid carrier are determined by its known composition  $c_{if}$ :

$$c_j\Big|_{x=0} = c_{jf} \,. \tag{7}$$

In describing the transfer process in the gas phase of the membrane contactor, let us assume that the composition of the gaseous mixture across the channel remains unaltered and obeys the laws of a noninteracting perfect gas. We shall also neglect the edge effects at the inlet into and outlet from the module. Under the assumptions being considered, the system of mass balance equations of the gaseous mixture components has the form [12, 20, 21]

$$\frac{d}{dx}\left[qg_{i} - \frac{P}{RT}g_{i}\sum_{j}D_{ij}\frac{dg_{j}}{dx}\right] = 2Hj_{ig-m},$$

$$\frac{dq}{dx} = 2H\sum_{i}j_{ig-m}.$$
(8)

The generalized diffusion coefficients  $D_{ij}$  are determined from the system [20, 21]

$$\sum_{k} \frac{g_{i}g_{k}}{\ddot{A}_{ik}} (D_{ij} - D_{kj}) = \delta_{ij} - \frac{M_{i}}{\overline{M}} g_{i} ,$$
$$\sum_{i} M_{i}g_{i}D_{ij} = 0 ,$$

here  $\overline{M} = \sum_{i} M_{i}g_{i}$  is the average molar mass of the mixture.

Let us write the momentum balance equation in a form convenient for determining the pressure distribution along the separation channel of the gas phase [13]:

$$\frac{dP}{dx} = \frac{3\eta RT}{Hb_g^3} \frac{q}{P}.$$
(9)

We restrict ourselves to the case of counterflow and assume the point of liquid feeding into the module to be the reference point on the longitudinal coordinate x = 0. The boundary conditions for Eqs. (8) and (9) are of the form

$$(qg_i)\Big|_{x=l} - \left(Pg_i\sum_j D_{ij}\frac{dg_i}{dx}\right)\Big|_{x=l} = q_fg_{if}, \quad \frac{dg_i}{dx}\Big|_{x=0} = 0, \quad P\Big|_{x=l} = P_f.$$

The binding conditions of the gas and liquid phases represent the values of the boundary flows of components of the liquid phase  $j_{jm-liq}$  and the gas phase  $j_{jg-m}$  for Eqs. (6) and (8). To this end, it is necessary to describe the phenomena proceeding at the gas-membrane and membrane-liquid interfaces. At the gas-membrane interface, N components of the gas phase dissolve by a particular mechanism in the membrane (or penetrate into the membrane pores) and diffuse towards the membrane-liquid interface, where, due to the sorption and chemical surface processes, the composition of the liquid phase changes, and some of the components can be formed or vanish.

Consider a particular case of small concentrations of sorbed components in the liquid carrier. Then the volumetric rate of flow, the density, and the transfer coefficients of the components are constant and the diffusion coefficients and the parameters of sorption isotherms are equal to their values in the component-carrier system. Let us also assume that there are no interconversions of substances in the system. Then the number of components-carriers in the liquid phase (components whose concentrations vary under sorption of the gaseous mixture) is equal to the number of gas-phase components and Eq. (1) can be written as

$$V\frac{\partial c_i}{\partial x} = D_i \frac{\partial^2 c_i}{\partial y^2}, \quad i = \overline{1, N}.$$
 (10)

In this case, the binding conditions are determined from the equations

$$j_{ig-m} = j_{im-liq} = Q_i \left( Pg_i - P_i^* (c_i \Big|_{y=0}) \right), \quad i = \overline{1, N}.$$
(11)

To decrease the number of investigated parameters of the process, let us assume that the influence of diffusion and the pressure gradient along the gas-phase channel can be neglected and the process of sorption of each component of the gas phase proceeds by Henry's law with the formation of one component in the liquid. The system of equations obtained under these assumptions was solved numerically with the use of grid methods.

We shall use the following similarity criteria in the investigations:  $Pe_i = V_{max}b_{liq}^2/(4D_il)$  — the Peclet number reflecting the ratio of the component convective flow along the liquid channel to the diffusion crossflow;  $B_i = Q_i K_i b_{liq}/(2D_i)$  — the penetrability ratio for the *i*th component of the membrane and the liquids;  $\varphi_i = LK_i/(q_f P_f)$  — the ratio between the liquid and gas flows;  $\chi_i = Q_i P_f S/q_f$  — the dimensionless ratio of the membrane penetrability to the feeding flow. It is easy to see that the ratio  $\chi_i$  is not an independent parameter and can be expressed in terms of  $\varphi_i$ ,  $B_i$ , and  $Pe_i$ .

We shall characterize the content of absorbed components in the liquid by the degree of saturation, defined as the ratio of the flow-averaged concentration of the component in the liquid to the value of its concentration in the liquid that is equilibrium with respect to the initial gaseous mixture:

$$\overline{L}_{i} = \frac{8}{3} \int_{0}^{1} \frac{c_{i}(\overline{x}, \overline{y}) K_{i}}{g_{if} P_{f}} \overline{y} (2 - \overline{y}) d\overline{y}.$$
(12)



Fig. 2. Degree of depletion of the mixture as to the WSC versus the ratio between the liquid and gas flows at various WSC penetrabilities: 1)  $B_1 = B_2 = \infty$ ; 2) 25; 3) 5; 4) 1; 5) 0.2. Pe<sub>1</sub> = Pe<sub>2</sub> = 2,  $K_2/K_1 = 30$ , and  $g_{1f} = 0.4$ .

Fig. 3. Degree of extraction of the PSC component versus the Peclet number at various penetrabilities of the PSC: 1)  $B_2 = 150$ ; 2) 25; 3) 5; 4) 1; 5) 1/6.  $B_1 = 5$ ,  $Pe_1 = Pe_2$ ;  $K_2/K_1 = 30$ ,  $g_{1f} = 0.4$ .

Consider the transfer process in the liquid phase at a constant composition of the gas phase ( $\varphi_i = 0$ ). This case is limiting and is realized due to either a small fraction of the gas flow being sorbed or the regime of complete mixing in the gas phase.

**Investigation of the Separation in a Membrane Contactor with a Physical Absorbent.** Let us restrict ourselves to the case of removal of one component from a binary mixture. Enumerate the components in decreasing order of solvability in the liquid carrier. The membrane absorber can perform the functions of cleaning the gaseous mixture from the well-sorbed component (WSC) and saturating the liquid with this component. An important characteristic therewith is the degree of extraction of the poorly sorbed component (PSC). Let us investigate the above characteristics depending on the dimensionless parameters of the process  $\chi$ , *B*,  $\varphi$ , and Pe.

Figure 2 shows the dependences of the WSC concentration on the dimensionless liquid flow  $\varphi_1$  at various values of  $B_1$ . With increasing  $\varphi_1$ , the values of  $g_1$  decrease and reach an arbitrary small value, and the difference between the curves thereby decreases with increasing  $B_1$  and already at  $B_1 > 5-10$  becomes small. This means that, on the one hand, at fixed properties of the liquid carrier, beginning with some value, the membrane penetrability does not influence the mass-transfer process, and, on the other hand, at a given membrane penetrability a decrease in the sorption capacitance of the liquid carrier with respect to the WSC, beginning with some value, also has a weak effect on the process. From Fig. 2 it is seen that practically complete removal of the WSC up to the value of  $g_{1f}/g_{1r} = 100$  at  $B_1 \ge 25$  and  $Pe_1 = 2$  is realized already at  $\varphi_1 = 3$ .

Note also that a change in the PSC penetrability at high values of  $B_1 > 5$  has a weak effect on the WSC concentration in the retentate flow  $g_{1r}$ . At small values of  $B_1 < 0.1-0.2$ , the influence of the membrane selectivity, beginning with some value of  $\chi_1/\chi_2 > 5-10$ , on the  $g_{1r}$  value is also insignificant. However, the degree of extraction of the PSC  $\psi_2$  can strongly depend on the membrane selectivity (Fig. 3). The degree of extraction is largely determined by the quantity of the component sorbed by the liquid. At small liquid flows the degree of extraction is close to unity independent of  $B_2$ , and at large flows the PSC penetrability produces a marked effect on the degree of extraction.

Figure 4 shows the dependences of the  $\varphi_{1\min}$  values characterizing the minimum liquid flow at which  $g_{1f}/g_{1r}$ = 100 (Fig. 4a) and the  $\chi_{1\min}$  values (Fig. 4b) characterizing, under the same conditions, the minimum ratio of the flow of the separated gaseous mixture to the membrane capacity on the Peclet number Pe. Actually, at given values of  $g_{1f}$  and  $g_{1f}/Ir = 100$  Fig. 4 reflects the universal relations between the process parameters that are independent of particular characteristics of the membrane, the carriers, and the MC geometry.

Interestingly, as  $B_1 \rightarrow 0$  the ratio of the liquid and gas flows  $\varphi_{1\min} \rightarrow \infty$  (Fig. 4a), which is equivalent (with the preservation of the interface penetrability) to the usual gas-separation membrane module with a low-pressure evacuated cavity. At the same time, the ratio of the membrane capacity to the gas flow  $\chi_{1\min}$  tends to a constant value (Fig. 4b). This value also corresponds to  $\chi_{1\min}$  for the gas-separation membrane module with vacuum in the low-pressure



Fig. 4. Minimum ratio between the liquid and gas flows for removing the WSC (a) and required ratio of the membrane capacity to the feeding flow of the gaseous mixture as to the WSC (b) versus the Peclet number for various membrane penetrabilities as to the WSC: 1)  $B_1 = 100$ ; 2) 25; 3) 1; 4) 0.2; 5) 0.04.  $B_1 = B_2$ ,  $Pe_1 = Pe_2$ ,  $K_2/K_1 = 30$ , and  $g_{1f} = 0.4$ 

Fig. 5. Outlet WCS concentration versus the inlet one in the gas phase at various dimensionless ratios between the liquid and gas flows as to the WSC: 1)  $\varphi_1 = 0.5$ ; 2) 0.95; 3) 1.05; 4) 1.2; 5) 1.5; a)  $\xi = 0.7$ ; b) 0.5; c) 0.3; d) 0.1.  $B_{g,liq1} = B_{g,liq2} = 5$ ; Pe<sub>1</sub> = Pe<sub>2</sub> = 0.5;  $K_2/K_1 = 30$ .

cavity. Thus, for these conditions the limiting stage of the process is the gas transfer through the membrane. The decrease in  $\chi_{1\min}$  with increasing Peclet number at given values of  $B_1$  is due to the relation between  $\chi_{1\min}$ ,  $B_1$ ,  $\phi_{1\min}$ , and Pe<sub>1</sub>.

The investigation of the membrane absorber operation at various compositions of the mixture being separated is also interesting. Figure 5 shows the dependences of the concentration of the component removed from the mixture  $g_{1r}$  in the retentate flow on its concentration  $g_{1f}$  in the feeding flow. The graphs were plotted at constant Pe for various values of  $\varphi_1$ . The figures also show the lines of constant values of the degrees of separation of the flow  $\xi = g_r/g_f$ . It is seen that the dependences at  $\varphi_1$  equal to some value of  $\varphi_{cr}$  change the character, and the value of  $\varphi_{cr}$  thereby is the larger, the higher the value of Pe<sub>1</sub>. At  $\varphi_1 < \varphi_{cr}$ , the WSC concentration in the retentate flow  $g_{1r}$  increases monotonically with increasing concentration in the feeding flow  $g_{1r}$  and tends to unity as  $g_{1f} \rightarrow 1$ . In the case where  $\varphi_1 > \varphi_{cr}$ , the WSC concentration in the retentate flow  $g_{1r}$  passes through the maximum. This means that at fixed parameters of the process an increase in the WSC concentration  $g_{1f}$  to a certain value of  $g_{1f,max}$  leads to a decrease in  $\xi$  and in  $g_{1r}$  to zero. Thus,  $g_{1f} = g_{1f,max}$  corresponds to complete absorption of the gas feeding flow by the liquid. The value of  $g_{1f,max}$  is the closer to unity, the closer  $\varphi_1$  to  $\varphi_{cr}$ . An increase in  $g_{1f}$  to a value higher than the fixed value of  $g_{1f,max}$  at a constant pressure in the gas phase P is only possible at the cost of increasing the gas flow  $q_f$  or decreasing the liquid flow L, and in both cases thereby a decrease in the value of  $\varphi_1 = LK_1/(q_f/P_f)$  occurs.

Figure 6 gives the distributions along the channel of the WSC concentration in the gas phase (Fig. 6a), the partial WSC flow in the gas phase (Fig. 6b), the derivative of the partial WSC flow in the gas phase (Fig. 6c), the WSC concentration in the liquid (Fig. 6d), and the PSC concentration in the liquid (Fig. 6e) at various WSC concen-



Fig. 6. Distributions along the channel of the WSC concentration in the gas phase (a), of the partial WSC flow in the gas phase (b), of the partial WSC flow derivative in the gas phase (c), of the WSC concentration in the liquid (d), and of the PSC concentration in the liquid (c) at various WSC concentrations at the inlet into the MC: 1)  $g_{1f} = 0.2$ ; 2) 0.5; 3) 0.9.  $B_1 = B_2 = 5$ , Pe<sub>1</sub> = Pe<sub>2</sub> = 0.5,  $\varphi_1 = 1.2$ , and  $\varphi_2 = 0.04$ .

trations at the inlet into the MC. At small values of the WCS concentration, the partial pressure of this component varies as the partial flow. This leads to a decrease in the motive force of mass transfer and, accordingly, in the derivative of the partial flow (Fig. 6d) practically by the linear law as the gaseous mixture moves from the inlet (x = 1)to the outlet (x = 0). An increase in the inlet WSC concentration leads to an increase in the sorbed flow density (Fig. 6d) in the vicinity of the beginning of the module (x is close to zero). However, in this region the WSC concentration for the investigated case weakly depends on the coordinate (Fig. 6a) at high values of  $g_{1f}$ , since the total flow at large concentrations of the WSC is largely determined by its partial flow. In the case of significant values of  $g_{1f}$  (for a high degree of cleaning from the WSC), the form of the distribution of the WSC concentration in the gas phase makes it possible to conventionally break down the contactor into three zones (Fig. 6a). In zone I, near the inlet the composition of the gaseous mixture changes only slightly, and the density of the sorbed WSC flow increases, which is due to the decrease in the liquid saturation (Fig. 6c) for the case of counterflow. In zone II, the major (considerable) part of the partial WSC flow has been sorbed by the liquid and a sharp decrease in the concentration begins due to the small total gas flow, which leads to the passage through the maximum of the sorbed flow density. In zone III, the WSC concentration in both phases is low and mainly fine recleaning of the gaseous mixture from the WSC (Fig. 6a) and sorption of the PSC (Fig. 6e), whose concentration  $g_2$  in the gas phase in this zone is close to unity, occur. The PSC sorbed in zones III and II of the contactor is partially desorbed in zones II and I, due to which circulation of the PSC in the contactor is realized. If, in zone III, complete PSC-saturation of the liquid occurs and in zones I and II PSC desorption is absent, then the degree of extraction for this component is



Fig. 7. WSC concentration in the retentate flow versus its concentration in the liquid at the outlet at various Peclet numbers in the cases of forward (a) and backward (b) flows: 1) Pe = 10; 2) 2; 3) 1; 4) 0.5; 5) 0.1.  $B_1 = B_2 = 50$ , Pe<sub>1</sub> = Pe<sub>2</sub>,  $K_2/K_1 = 30$ , and  $g_{1f} = 0.4$ .

$$\psi_{2\min} = \begin{cases} 1 - \frac{\varphi_2}{g_{2f}}, & \frac{\varphi_2}{g_{2f}} < 1 \\ 0, & \frac{\varphi_2}{g_{2f}} \ge 1 \end{cases}$$

;

The presence of the PSC desorption zone distinguishes the MC from the conventional gas-separation module and makes it possible to considerably increase the degree of extraction of the PSC even under conditions where the value of  $\varphi_2/g_{2f}$  characterizing the dimensionless inlet ratio of the liquid flow to the partial flow of the PSC is greater than one.

From Fig. 6e it is seen that practically it is possible to realize regimes where the degree of saturation as to the PSC is higher than one (with respect to the composition of the initial gaseous mixture). To reduce PSC losses, it is necessary to reduce the zone of effective sorption of this component (zone III), e.g., by decreasing the membrane area or the liquid carrier flow. The calculations have shown that a decrease in the area by the value corresponding to zone III leads to an increase in the outlet PSC concentration by up to 1% and in the degree of extraction from 78 to 90%. At WSC inlet concentrations  $g_{1f} < 20\%$ , the degree of saturation of the liquid as to the PSC cannot be much higher than unity because of the weak change in the PSC concentration in the gaseous phase of the contactor. In this case, the degree of extraction as to the PSC is bounded below by  $\psi_{2min} \sim 1 - \varphi_2$ .

Let us analyze the influence of the organization of the hydrodynamic flow on the efficiency of the process of cleaning the gaseous mixture from the WSC. We shall consider two cases: forward flow and backward flow. Figure 7 presents the dependences of the WSC concentrations in the retentate flow on the dimensionless concentration of this component in the liquid at the outlet from MC in the case of forward (Fig. 7a) and backward (Fig. 7b) flows at various values of Peclet numbers. From Fig. 7a it is seen that in the case of forward flow at small Pe values the dimensionless concentrations at the outlet from the MC are practically equal. This means that at sufficiently small Peclet numbers and high penetrabilities of membranes towards the outlet from the module there occurs practically complete flattening of the WSC concentration profile at a level close to equilibrium. As the Peclet number increases, the degree

of saturation of the liquid decreases, which shows up as a deviation of the dependences from the equilibrium ones:  $g_{1r} = L_1$ .

It can easily be seen that to provide a high degree of cleaning of the gaseous mixture from the WSC\_ $(g_{1r} \approx 0)$  under forward flow conditions, it is essential that the degree of saturation of the liquid be close to zero  $(L_1 \sim 0)$ , i.e., it is necessary to ensure an infinite liquid flow  $(\varphi_1 \rightarrow \infty)$ .

A different situation takes place in the case of backward flow. From Fig. 7b it is seen that for a high cleaning from the WSC a finite liquid flow is needed (the outlet concentration in the liquid has a certain value at a concentration in the gas close to zero for each Peclet number). The required liquid flow for close to complete WSC removal can be determined by means of Fig. 4a.

It is seen that the degree of saturation (Fig. 7b) at which practically complete removal of the WSC is realized  $(g_{1f}/g_{1r} = 100)$  decreases with increasing Peclet number. At small values of Pe (Pe < 0.1), the degree of saturation becomes close to unity. Thus, at small Peclet numbers in the case of backward flows, conditions close to ideal ones, under which a high degree of cleaning from the WSC in a liquid flow close in size to the least possible one, which is able to absorb the whole WSC ( $\varphi_1 = 1$ ), are realizable. The degree of PSC extraction in this case is determined by the sorption selectivity of components in the liquid and can reach very high values.

# CONCLUSIONS

1. A mathematical model of the process of selective mass transfer in a plane-frame membrane absorber (contactor) has been developed. A numerical study of the process of cleaning a gaseous mixture has been made with the example of the physical mechanism of sorption of gases in a liquid carrier.

2. Similarity criteria for the considered membrane contactors have been formulated.

3. The range of values of regime parameters in which limited penetrability of the membrane has no significant influence on the mass transfer has been determined.

4. The influence of the composition of the initial mixture on the cleaning process in the MC has been investigated. It has been shown that an increase in the WSC concentration in the feeding flow under certain conditions leads to its decrease in the retentate flow.

5. It has been established that an excess area of the membrane upon removal of the WSC leads to a decrease in the degree of extraction of the PSC.

6. Comparison of the forward-flow and backward-flow operating conditions of the MC has been made. It has been shown that in the case of forward flow the regime of complete removal of the WSC is impossible, whereas the backward-flow conditions permit cleaning at relatively small liquid flows bounded below by the WSC solvability in the liquid.

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# **NOTATION**

 $a_j$ , activity of the *j*th component in the liquid phase, mole/m<sup>3</sup>;  $b_g$  and  $b_{liq}$ , gap thickness of the gas and liquid channels, m;  $B_i = \frac{Q_i K_i b_{liq}}{2D_i}$ , penetrability ratio for the *i*th component of the membrane and the liquid layer;  $c_j$ , concentration of the *j*th component in the liquid phase, mole/m<sup>3</sup>;  $\overline{c_j} = \frac{c_j K_j}{g_{jf} P_f}$ , dimensionless concentration of the *j*th component in the liquid;  $D_j$ , diffusion coefficient of the *j*th component in the liquid phase, m<sup>2</sup>/sec;  $D_{ij}$ , generalized diffusion coefficients, m<sup>2</sup>/sec;  $g_i$ , concentration of the *i*th component in the gas phase;  $\overline{g_i} = g_i/g_{if}$ , relative concentration of the *i*th component in the gas phase; *H*, width of the liquid channel, m;  $j_{ig-m}$ , density of the membrane-sorbed flow of the *i*th component of the gaseous mixture, mole/(sec·m<sup>2</sup>);  $j_{km-liq}$ , flow density of the *k*th component realized in the liquid at the membrane–liquid interface, mole/(sec·m<sup>2</sup>);  $K_i$ , Henry constant of sorption of the *i*th component of the gaseous mixture in the liquid, Pa·m<sup>3</sup>/mole; *l*, liquid channel length, m; *L*, volumetric rate of liquid flow, m<sup>3</sup>/sec;  $\overline{L}_{i} = \frac{8}{3} \int_{0}^{1} \overline{c_{i}(x, y)} \cdot \overline{y}(2 - \overline{y}) d\overline{y}, \text{ degree of liquid saturation as to the$ *i* $th component; <math>M_{i}$ , molar mass of the *i*th component, mole/kg;  $\overline{M} = \sum M_{i}g_{i}$ , average molar mass of the mixture at a point, mole/kg; N, number of gas-phase components;

*P*, pressure of the gaseous mixture, Pa;  $P_i^*(c_i)$ , sorption isotherm of the *i*th component of the gaseous mixture in the liquid, Pa; Pe<sub>i</sub> =  $\frac{V_{\text{max}}b_{\text{liq}}^2}{4D_i l}$ , Peclet number with respect to the *i*th component;  $Q_i$ , membrane penetrability for the *i*th component of the gaseous mixture, mole/(sec·Pa·m<sup>2</sup>); *q*, gaseous mixture flow, mole/sec;  $\overline{q_i} = \frac{g_i q}{q_f}$ , dimensionless partial flow of the *i*th component; R = 8.31 J/(mole·K), universal gas constant; S = 2Hl, membrane area, m<sup>2</sup>; *T*, process temperature, K; *V*, liquid flow velocity, m/sec; *x*, longitudinal coordinate of the liquid phase channel, m;  $\overline{x} = x/l$ , dimensionless longitudinal coordinate,  $0 \le \overline{x} \le 1$ ; *y*, transverse coordinate of the liquid phase channel, m;  $\overline{y} = 2y/b_{\text{liq}}$ , dimensionless transverse coordinate,  $0 \le \overline{y} \le 1$ ; *Z*, number of liquid phase components;  $\chi_i = Q_i P_f S/q_f$ , dimensionless ratio of the membrane penetrability to the feeding flow;  $\delta_{ij}$ , Kronecker symbol;  $\eta$ , dynamic viscosity of the gaseous mixture, Pa·sec;  $\varphi_i = \frac{LK_i}{q_f P_f}$ , dimensionless ratio between the liquid and gas flows;  $\sigma_i$ , source function, mole/(m<sup>2</sup>·sec);  $\xi = q_r/q_f$ , degree of flow separation;  $\psi_i$ , degree of extraction as to the *i*th component in the retentate flow, %;  $D_{ik}$ , binary diffusion coefficients, m<sup>2</sup>/sec. Subscripts: liq, liquid; g, gas; m, membrane; f, feeding; r, retentate; max and min, minimum and maximum values; cr, critical.

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